MAT336 HOMEWORK 6 (DUE APRIL 6, 2016)

Please write your name, student number and *your section* (0101 or 2001) in every page that you turn in. The section code is important in order to help us upload your grade in Blackboard (each section has a different homepage). The more time we save doing this, the more questions we can correct and the more feedback you will have!

If you turn the assignment in (with most of the questions done) you will get 50%. We will partially mark the questions and the other 50% of the grade depends on the mark obtained on Problem 1, Sec 6.3 K, 8.1 A.

Problems:

Problem 1. Define:

$$S_n(x) = \sum_{k=1}^{n} \frac{sin^k(x)}{k}$$

And consider the function

$$f(x) = \lim_{n\to\infty} S_n(x) = \sum_{k=1}^{\infty} \frac{sin^k(x)}{k}$$

for any $x \in \mathbb{R}$ where the series converges.

(a) Determine all $x \in \mathbb{R}$ for which $f(x)$ converges, and all $x \in \mathbb{R}$ where it diverges. Justify your answer.

(b) Prove that S_n converges uniformly to f on $[0, \frac{\pi}{4}]$ (Hint: read section 8.4 and use one of the results in there).

(c) Prove that S_n does not converge uniformly to f on $[0, \frac{\pi}{2})$ (Note: you don't need the results from Section 8.4 for this exercise - the definition of uniform convergence is enough).

From the book, you should turn in the following problems:

- Section 6.3: F, H, K, O
- Section 6.4: C, D
- Section 8.1: A, B, C

Problems which you don't need to turn in on April 6 anymore:

- Section 8.2: F and Section 8.4: C. Reason: we are one class behind the schedule and the necessary topics will only be covered on Monday. Don't worry about the "behind the schedule" part, since the last two hours of the course were supposed to be "Catching up/Applications", so one of these hours will be "Catching up". We will not rush the material!