MAT336 HOMEWORK 5 (DUE MARCH 23, 2016)

Please write your name, student number and *your section* (0101 or 2001) in every page that you turn in. The section code is important in order to help us upload your grade in Blackboard (each section has a different homepage). The more time we save doing this, the more questions we can correct and the more feedback you will have!

If you turn the assignment in (with most of the questions done) you will get 50%. We will partially mark the questions and the other 50% of the grade depends on the mark obtained on Problem 2, 3 and Section 5.4 K, .

Problems:

Problem 1. Suppose $\mathcal{A}$ is any indexing set, and $\{A_\alpha \mid \alpha \in \mathcal{A}\}$ is a collection of sets where each $A_\alpha \subset X$ is connected. If there exists at least one point $a \in \bigcap_{\alpha\in\mathcal{A}} A_\alpha$, then show that $\bigcup_{\alpha\in\mathcal{A}} A_\alpha$ is connected.

Problem 2. Show that the closure of any connected set is also connected.

Problem 3. Consider the function

$$f(x) = x|x - a|$$

where a is a fixed constant in $\mathbb{R}$. Sow that f is differentiable over every point in $\mathbb{R}$ if and only if $a = 0$.

You will need the following definition for some of the problems in the textbook.

Definition 1. A real valued function f is differentiable on $[a, b] \subset \mathbb{R}$ if f is differentiable in the usual sense on (a, b), and the left derivative $\lim_{x\to b^-} \dfrac{f(x) - f(b)}{x - b}$, and the right derivative $\lim_{x\to a^+} \dfrac{f(x) - f(a)}{x - a}$ both exist.

Then the notation $f \in C^k[a, b]$ (seen in the textbook problems) means that f is differentiable k times on $[a, b]$, and $\frac{d^k f}{dx^k}$ is a continuous function on $[a, b]$.

From the book, you should turn in the following problems:

- Section 5.3: K
- Section 5.4: B, K
- Section 5.6: D, E, I
- Section 6.1: D, S
- Section 6.2: A, G