MAT336 HOMEWORK 3 (DUE FEB. 24, 2016)

Please write your name, student number and *your section* (0101 or 2001) in every page that you turn in. The section code is important in order to help us upload your grade in Blackboard (each section has a different homepage). The more time we save doing this, the more questions we can correct and the more feedback you will have!

If you turn the assignment in (with most of the questions done) you will get 50%. We will partially mark the questions and the other 50% of the grade depends on the mark obtained on Problems 1, 2 and Sec 2.7 H.

Problems:

Problem 1. Show that if $\{a_{n_k}\}_{k\in\mathbb{N}}$ is a convergent subsequence of a bounded sequence $\{a_n\}_{n\in\mathbb{N}}$, then

$$\lim_{k\to\infty} a_{n_k} \le \limsup_{n\to\infty} a_n.$$

Remark: It is actually true that:

$$\liminf_{n\to\infty} a_n \le \lim_{k\to\infty} a_{n_k} \le \limsup_{n\to\infty} a_n.$$

but you don't need to prove the second inequality.

Problem 2. Show that if $\sum_{i=1}^{\infty} |a_i|$ and $\sum_{i=1}^{\infty} |b_i|$ are convergent series, then $\sum_{i=1}^{\infty} a_i\, b_i$ is convergent.

From the book, you should turn in the following problems:

- Section 2.6: J
- Section 2.7: H (Hint: argue by contradiction)
- Section 2.8: A, B (hint: think about series), E
- Section 3.1: D
- Section 3.2: G, N (Hint: the fact that f is decreasing is crucial. Use the geometrical interpretation of integral in order to use the hint from the book), O